Advanced Placement

PHYSICS C

2007 Feb 12 Mon

$x \rightarrow \theta$

$v \rightarrow \omega$

$a \rightarrow \alpha$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$\downarrow$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$v = r\omega$

$a_T = r\alpha$

$(a_c = \omega^2 r)$

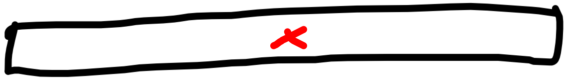
$$K = \sum_{\substack{\text{all} \\ \text{particles}}} \frac{1}{2} m_i v_i^2 = \sum_{\substack{\text{all} \\ \text{particles}}} \frac{1}{2} m_i (r_i \omega)^2$$

$$= \frac{1}{2} \Big(\sum_{\substack{\text{all} \\ \text{particles}}} m_i r_i^2 \Big) \omega^2$$

$$= \frac{1}{2} I \omega^2$$

rotational inertia

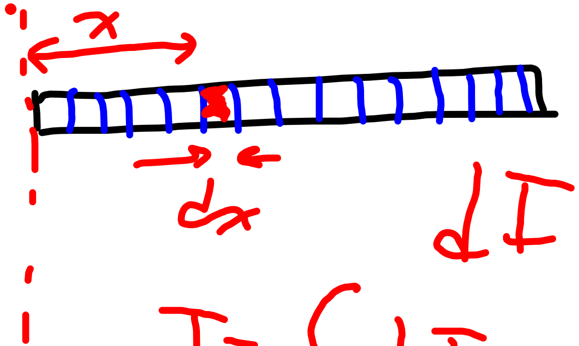
(= moment of inertia)



$M = 1\ \text{kg}$

$L = 1\ \text{m}$

$$I = M\left(\tfrac{1}{2}L\right)^2$$



$$\frac{dm}{M} = \frac{dx}{L}$$

$$dm = \frac{M}{L}dx$$

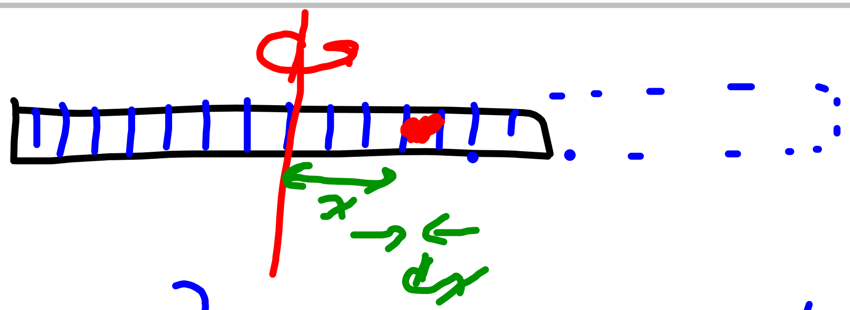
$$dI = dm\ x^2$$

$$I = \underbrace{\int}_{\text{ruler}} dI = \underbrace{\int}_{\text{ruler}} dm\ x^2$$

$$= \int_0^L \frac{M}{L} dx\, x^2 = \frac{M}{L}\int_0^L x^2 dx = \frac{M}{L}\frac{x^3}{3}\Big|_0^L$$

$$\tfrac{1}{3}ML^2$$

$$\parallel$$



$$dI = dm\, x^2$$

$$I = \int_{\text{whole ruler}} dI = \int dm\, x^2 = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{M}{L}\, dx\, x^2$$

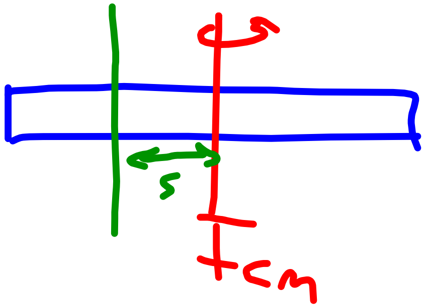
$$x^3\Big|_0^{\frac{L}{2}} = \left(\frac{L}{2}\right)^3 - 0^3$$

$$= 2\int_0^{\frac{L}{2}} \frac{M}{L}\, dx\, x^2 = 2\frac{M}{L}\, \frac{x^3}{3}\Big|_0^{\frac{L}{2}}$$

$$= \frac{2}{3} M L^2 \left(\frac{1}{8} - 0\right) = \frac{1}{12} M L^2$$

Parallel Axis Thm

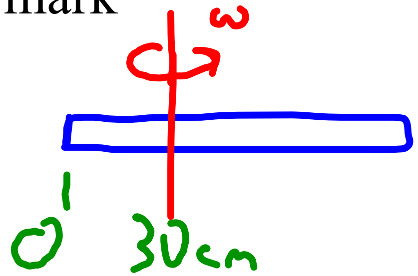
$$I = I_{cm} + M(s^2)$$

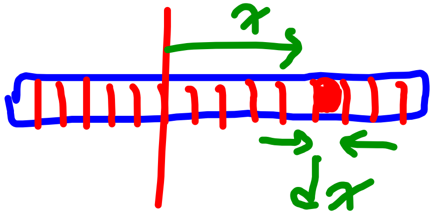


Advanced Placement
PHYSICS C

2007 Feb 13 Tue

Find the rotational inertia of a 0.91 kg meterstick about a vertical axis through the 30 cm mark
(a) by direct integration
(b) by Parallel Axis Theorem





$$I = \int dI$$

$$= \int dm\, x^2 = \int_{-0.3}^{0.7} \frac{M}{L} dx\, x^2$$

$$= \frac{M}{L} \left.\frac{x^3}{3}\right|_{-0.3}^{0.7} = \frac{0.91\,\text{kg}}{3(1\text{ m})}\left[(0.7\text{m})^3 - (-0.3\text{m})^3\right]$$

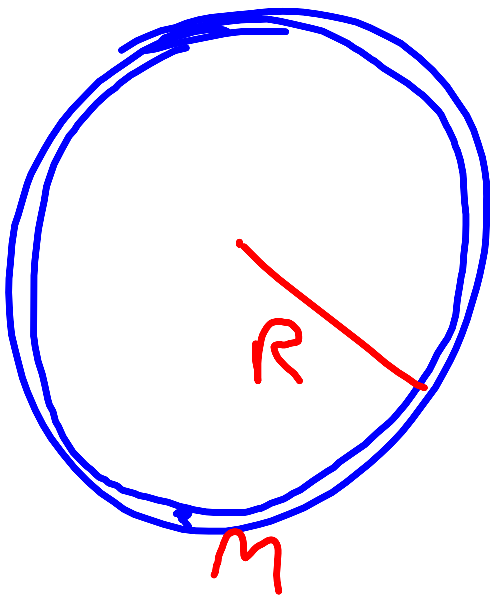
$$= 0.112\ \text{kg m}^2$$

$$I = I_{cm} + Ms^2$$

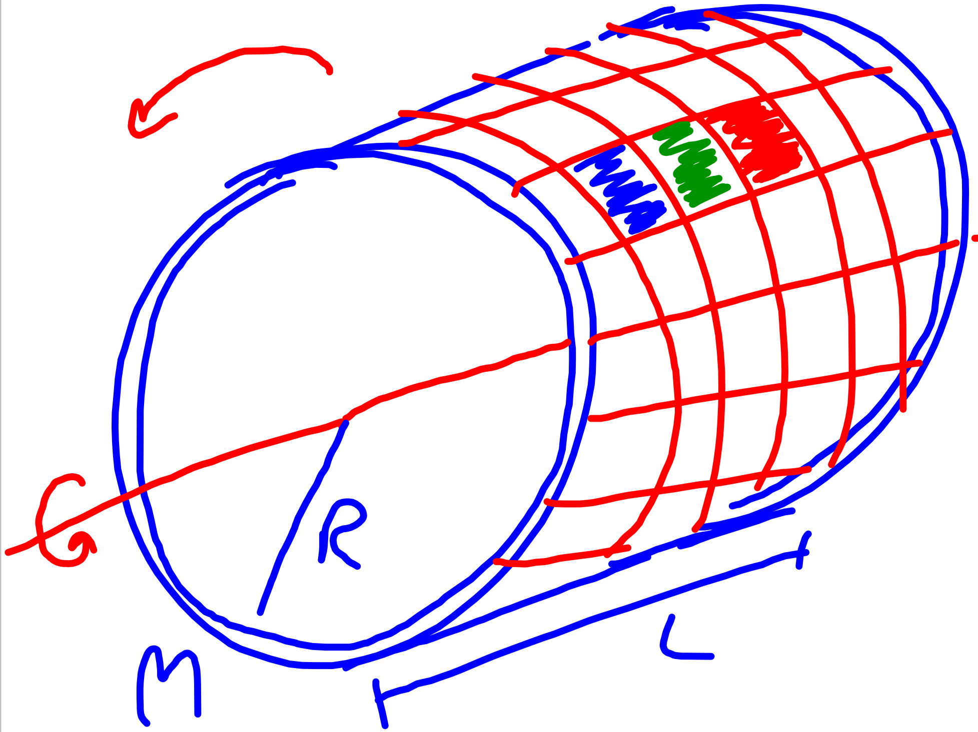
$$= \frac{1}{12}ML^2 + Ms^2$$

$$= \frac{1}{12}(0.91\,kg)(1\,m)^2 + 0.91\,kg\,(0.2\,m)^2$$

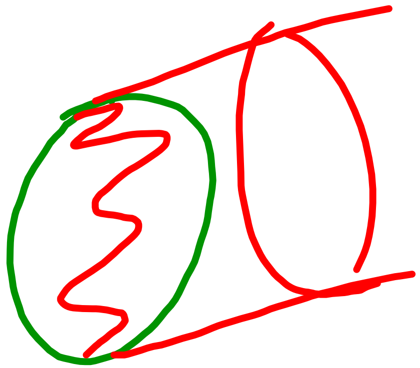
$$\rightarrow = 0.112\ kg\,m^2$$



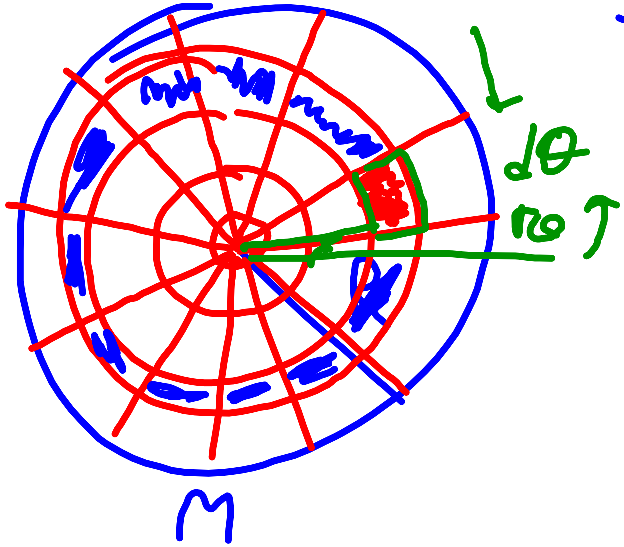
$$I = \int dI$$
$$= \int dm\, R^2$$
$$= R^2 \int dm$$
$$= MR^2$$

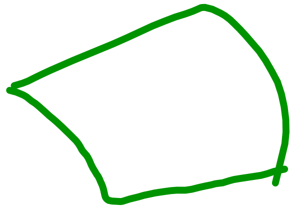


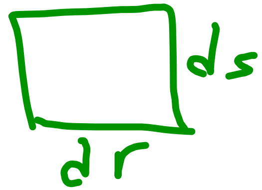
$I =$



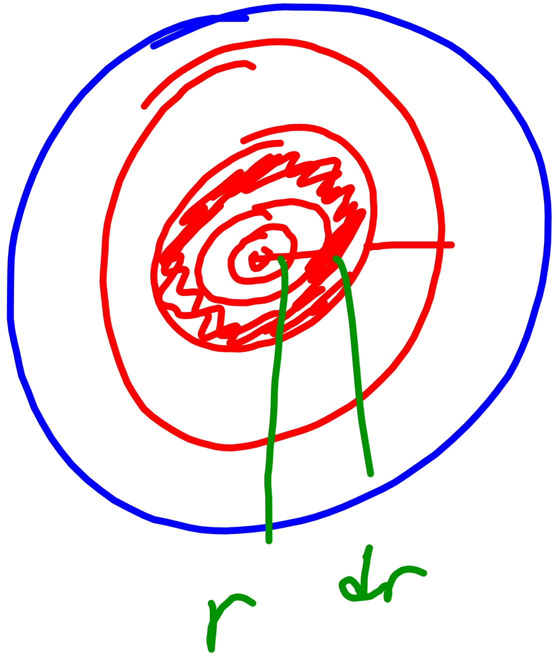
$I = ?$







$$dr\ ds$$
$$= dr\ r\,d\theta$$
$$= r\,dr\,d\theta$$



$$I = \int dI = \sum \text{rings}$$

$$= \int dm\, r^2$$

$$dI = dM\, R^2$$

$$\frac{dm}{M} = \frac{dA}{A}$$

$$\frac{dM}{dA} = \frac{M}{A}$$