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Finding an equation for the velocity of a chain falling off the edge of a table.

Abstract:
In this lab, the test was to compare a pysically derived equation to the results obtained from a performed experiment. Finding the instantaneous velocity is difficult to calculate because the frictional force at any given point in time of the chain system changes. Because the mass of the chain that interacts with the surface track is not constant, Fn is not constant. Using the equation Ff = μFn, we needed an to calculate μ. Knowing μ, we plugged that into a derived equation and calculated velocity with respect to the length of the chain hanging off of the table. After nuerous calculations and tedious video analysis, we found that as the length of the chain hanging off the table increases, the velocity of the chain also increases.

Theory:

Given θ (the angle between level and the cart track), m, and a, we can solve for μ:

$F_{net} = F_{gx} - F_n$

$ma = F_g * \sin\theta - (F_g * \cos\theta)\mu$

$a = g * \sin\theta - \mu * g * \cos\theta$

$$\mu = \frac{(g * \sin\theta - a)}{(g * \cos\theta)}$$

Knowing μ, m, L, Xo, E=Wnc D=m / L m= Dx X= L - x, we can find an expression for velocity of a chain falling off a table:

$E_0 = m_0 * g * y_{h0} \quad y_h = ½\, x_0$

$E_0 = D * x_0 * g * (-x_0/2) = -½\, D * g * x_0^2$

$E = U_g + KE$

$E = -½\, D * g * x^2 + ½\, D * x * v^2$

$W_{nc} = \int F * dX = \int -\mu * D * X * g * dX$

$\quad = -\mu * D * g * \int X * dX$

$\quad = -\mu * D * g * (½\, X^2)$

$E - E_0 = W_{nc}$

$-½ * D * g * X^2 + ½ * D * x * v^2 - (-1/2 * D * g * X_0^2) = -\mu * D * g * ((L - X_0)^2/2)$

$-g * x^2 + x * v^2 + g * X_0^2 = -\mu * g * (L - X_0)^2$

and x=L at E, so...

$-g * L^2 + L * v^2 + g * X_0^2 = -\mu * g * (L - X_0)^2$

$$V^2 = \frac{(g * L^2 - g * X_0^2 - \mu * g * (L - X_0)^2)}{L}$$

Equipment: (Include picture of the setup for solving for MEW from the shared folder)

Logger Pro
Table
Chain
Stopwatch
Meter Stick
Cart Track

Ring Stand
Calculator

Procedure:

First the coefficient of friction, μ, must be calculated. To do this you must slide the chain down the track while the track is on an incline.

Setup the cart track on an incline. For ease, we used a ring stand, which coincidentally linked well to the cart track to form an angle off of the table top.

Then, if not already displayed on the cart track, measure the distance that the chain will cover after being released. Since our track had metric units of length already labeled, we dropped the chain from 0 cm: the top of the track.

Then record all of the rudimentary data. This would include the mass and length of the chain, in addition to the angle at which the track lays off of the table top, or whichever surface the cart track lay on. Also, measure the x component and y component of the track's inclined length. Since the track forms a right triangle, you can use the sides of the formed triangle to calculate the angle between the track and the surface which it rests on.

Then, the experiment is ready to be performed. Have a stopwatch ready to record the time it will take the chain to travel from the top of the track to the bottom of the track. It is important that you place the chain in a ridge in the track so it has little if any space to change shape. If the chain curls or folds, the trial should be thrown out, since the transformation of the shape of the chain will yield variances in frictional forces.

Place the chain at the top of the track, or wherever it is that you plan to begin the drop from. When the chain is released, start the stopwatch and record the time until the chain reaches the end of the track. When the chain reaches the end of the track, pause the stopwatch and record the time taken to descend the path.

Since only taking one trial would be inaccurate, perform the experiment several times. According to the law of large numbers, theoretically, the more trials that you perform, your average time will approach the accepted value.

Data/Analysis:

Using a high speed camera and Logger Pro software, we were able to analyze the motion of the chain off a table. We found graphs of the position versus time, and Logger Pro's calculated velocity versus time (because it was filmed in high-speed, the scale of time had to be adjusted by a factor of one seventh). The four digit numbers indicate the different LoggerPro file names.

X Velocity vs. True Time:

3158: $y= 1.267x^2 – 0.2583x + 0.02653$

3159: $y= 1.053x^2 – 0.2564x + 0.02403$

Data from the Incline:

Mass of chain: 3.44 *g*

Length of chain: 37.8 *cm*

θ: 20.39°

Length of track: 122 *cm*

Times:

- Trial 1: 1.28 s
- Trial 2: 1.22 s
- Trial 3: 1.28 s
- Trial 4: 1.25 s
- Trial 5: 1.22 s
- Trial 6: 1.25 s
- Trial 7: 1.22 s

By plugging into our equation for we got: μ= .273

Xo= 0.127m

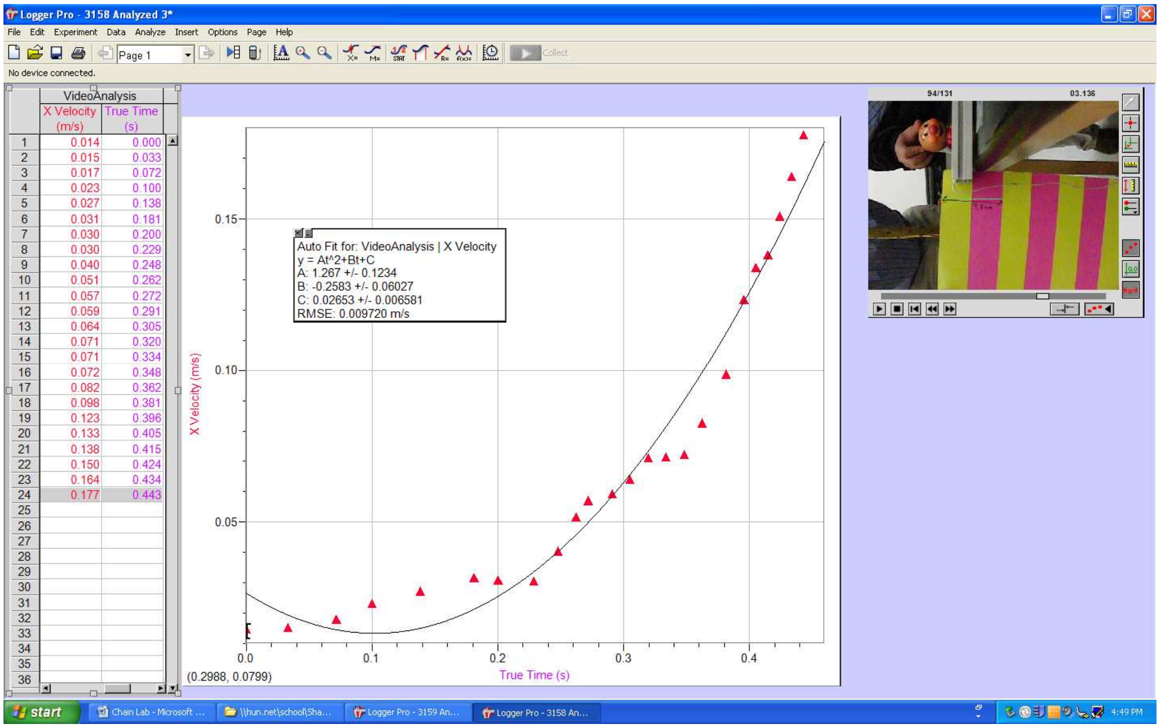
$g=9.8 m/s^2$

$$v^2 = \frac{g*L^2 - g*Xo^2 - \mu*g*(L-Xo)^2}{L}$$

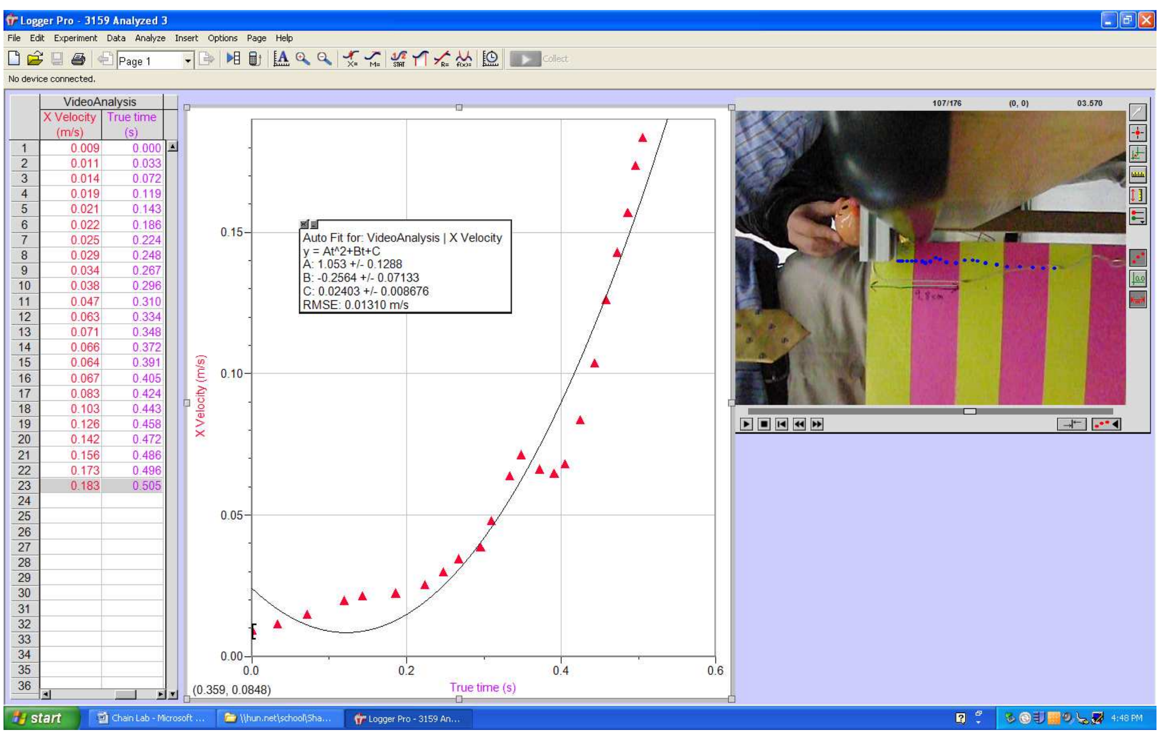
v= 1.685m/s (theoretical value)

See Graphs and Tables next page:

3158:


Logger Pro - 3158 Analyzed 3*
VideoAnalysis
X Velocity (m/s) | True Time (s)
0.014 0.000
0.015 0.033
0.017 0.072
0.023 0.100
0.027 0.138
0.031 0.181
0.030 0.200
0.030 0.229
0.040 0.248
0.051 0.262
0.057 0.272
0.059 0.291
0.064 0.305
0.071 0.320
0.071 0.334
0.072 0.348
0.082 0.362
0.098 0.381
0.123 0.396
0.133 0.405
0.138 0.415
0.150 0.424
0.164 0.434
0.177 0.443
Auto Fit for: VideoAnalysis | X Velocity
y = At^2+Bt+C
A: 1.267 +/- 0.1234
B: -0.2583 +/- 0.06027
C: 0.02653 +/- 0.006581
RMSE: 0.009720 m/s
X Velocity (m/s)
True Time (s)
(0.2988, 0.0799)

3159:


Logger Pro - 3159 Analyzed 3
VideoAnalysis
X Velocity (m/s) | True time (s)
0.009 0.000
0.011 0.033
0.014 0.072
0.019 0.119
0.021 0.143
0.022 0.186
0.025 0.224
0.029 0.248
0.034 0.267
0.038 0.296
0.047 0.310
0.063 0.334
0.071 0.348
0.066 0.372
0.064 0.391
0.067 0.405
0.083 0.424
0.103 0.443
0.126 0.458
0.142 0.472
0.156 0.486
0.173 0.496
0.183 0.505
Auto Fit for: VideoAnalysis | X Velocity
y = At^2+Bt+C
A: 1.053 +/- 0.1288
B: -0.2564 +/- 0.07133
C: 0.02403 +/- 0.008676
RMSE: 0.01310 m/s
X Velocity (m/s)
True time (s)
(0.359, 0.0848)

<u>X Velocity vs. True Time:</u>

3158: $y= 1.267x^2 - 0.2583x + 0.02653$

3159: $y= 1.053x^2 - 0.2564x + 0.02403$

3158: The time at which the string is just completely off is approximately 0.443s as appears on the table above.

$y= 1.267x^2 - 0.2583x + 0.02653= 0.150$

v_1=0.150m/s (Experimental value 1)

3159: The time at which the string is just completely off is approximately 0.505s as appears on the table above.

$y= 1.053x^2 - 0.2564x + 0.02403= 0.163$

v_2= 0.163m/s (Experimental Value 2)

<u>v_{avg}= 0.157m/s (Average Experimental Value)</u>

Conclusion:

As the analysis illustrates, we got an experimental average velocity of 0.157m/s, and the theoretical velocity was 1.685m/s. This is obviously a very high error. There are several sources of error. First, when the string falls off the cliff, the string shakes too violently to measure correctly. However, this is unavoidable due to the nature. Secondly, due to the violent shake, the LoggerPro itself cannot exactly analyze the video. We also had only a few data, but this does not seem to be affecting too much since the error is too high. Since the largest error bar is only +/- 0.1234, the procedure of the lab must be changed in order to get a better result.