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2
3 **...aaaaand there it goes**

4 **An investigation of the motion of a sliding chain**

5
6 **Abstract**

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8 We were supposed to find an expression for v , the velocity at the moment the last bit of the
9 chain fell off the table. We came up with the equation $v = \sqrt{(gL(1-(x/L)^2) - (0.25)(\mu)(1-$
10 $(x/L))^2)}$ mathematically under the authority of Mr. Gilroy, thus answering the ultimate
11 question of the lab. Using Logger Pro and a high-speed camera, we were able to analyze a
12 slow-motion video of the chain sliding off the table, and with the recorded data, verify that
13 the model is accurate within the calculated error bounds. This means it is a good model of
14 the chain's motion.

15
16 **Theory**

17 The master equation we used, $v = \sqrt{(gL(1-(x/L)^2) - (0.25)(\mu)(1-(x/L))^2)}$, is the result of a
18 lengthy derivation which will not be reproduced here. However the essential theory behind
19 our reasoning is that as the chain slides over the edge, more mass is added to the
20 overhanging mass, increasing the pull force of gravity. At the same time, less of the chain is
21 resting on the table, which makes the frictional force decrease. So as the chain slides, the
22 pulling force increases, and friction decreases in a rather complicated way. We were able to
23 create a sensible model equation using an energy-based approach, and set out to verify it
24 with hard data.

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26 **Equipment**

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28 Wooden board
29 Metal chains
30 Meter Rule
31 Camera with high-speed capability
32 Duct tape
33 Metal jack
34 Protractor

35
36 **Procedure**

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38 *To find the data for the coefficient of kinetic friction*

- 39
40 1. We placed a wooden board on a known angle θ from the horizontal using clamps and
41 duct tape to secure the board in position.
42 2. We then set the camera up so the front of the lens was parallel to the surface of the
43 board, so as not to incur any perspective distortion that would skew our video analysis.
44 3. We placed two marks on the board, 40 centimeters apart, to be later used as a scale in
45 the video analysis.

46 4. After placing the chain with known mass m_1 at the top of the board and releasing it, we
47 filmed it travelling down the board.

48
49 *To find the data for the final velocity*

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51 1. We removed the clamps from beneath the board and set the board on the edge of a desk.
52 2. We used sandpaper on the edge of the board to produce a more uniform edge for the
53 chain to slide off.
54 3. We set the camera up so the front of the lens was perpendicular to the path of the chain.
55 4. We released the chain and filmed the falling chain to use in later video analysis.

56
57 **Data/Analysis**

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59 *To compute the data for the velocity*

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61 In Logger Pro, we were able to measure the velocity of the end of the chain at the moment it
62 slides off the table via video analysis. Analysis of six trials with the same setup provided 24
63 points of final velocity data (four per trial), which was averaged together, to provide a final
64 value of -0.3747 m/s with a σ value of 0.0396.

65

Trial #	Value 1 (m/s)	Value 2 (m/s)	Value 3 (m/s)	Value 4 (m/s)
1	-0.402	-0.398	-0.419	-0.405
2	-0.341	-0.418	-0.327	-0.453
3	-0.292	-0.307	-0.386	-0.365
4	-0.322	-0.361	-0.371	-0.418
5	-0.367	-0.426	-0.401	-0.385
6	-0.359	-0.374	-0.362	-0.334

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67
68 The four values for each trial were from four consecutive frames taken at the very end of
69 the video, to provide the most balanced average for each trial. Using just one data point
70 increases problems due to various sources of error.

71
72 *To calculate a value for the kinetic friction coefficient.*

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74 Our strategy was to record a high-speed video of a chain segment sliding down a 30 degree
75 incline. Using Logger Pro, we were able to calculate the acceleration over a known distance,
76 then use forces to compute the force of friction, and since $F_f = \mu(mg)$, we were easily able to
77 obtain a value for μ_k .

78

v_i (m/s)	v_f (m/s)	a (m/s ²)	μ
0	-0.153	-0.023409	0.26764735169492
0	-0.145	-0.021025	0.2678998940678
0	-0.163	-0.026569	0.2673126059322
0	-0.157	-0.024649	0.26751599576271
0	-0.158	-0.024964	0.26748262711864
0	-0.154	-0.023716	0.26761483050847
		Average	0.2675788837
		σ value	0.0001794215

79

80 The equation we came up with to model this scenario is the product of a long derivation,
81 not reproduced here, but based on equations for energy and work. The final equation is

82

$$83 \quad v = \sqrt{(gL(1-(x/L)^2) - (0.25)(\mu)(1-(x/L))^2)}$$

84

85 Using the value of μ calculated from the video analysis, L (the total length of the chain), and
86 x_0 (the length of chain hanging off the table at the start), the predicted final velocity can be
87 calculated to be equal to: 2.426 m/s

88

89 **Conclusion**

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91 The predicted velocity of 2.426 m/s seems extraordinarily high when compared to the
92 measured speed of 0.3747 m/s. *LoggerPro* does not recognize the fact that the camera shot
93 the footage at 210 frames per second, as opposed to 30 and as a result, the velocity
94 measurements are 1/7th of their true value. Multiplying 0.3747 by 7 gives 2.6229 m/s with
95 an error of 0.2772 (also needed to be multiplied by 7). 2.426 falls well within the range of
96 velocities: 2.3457-2.9001. This means the model is an accurate model of the chain's motion.