HONORS PHYSICS

L140-008B : Theory

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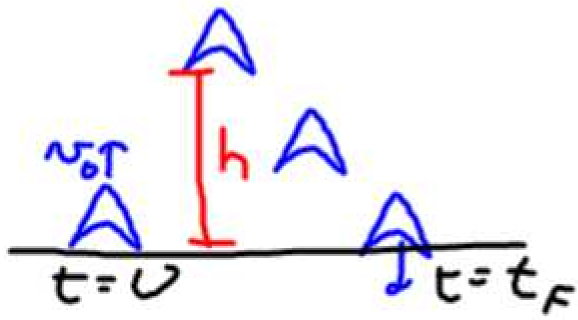
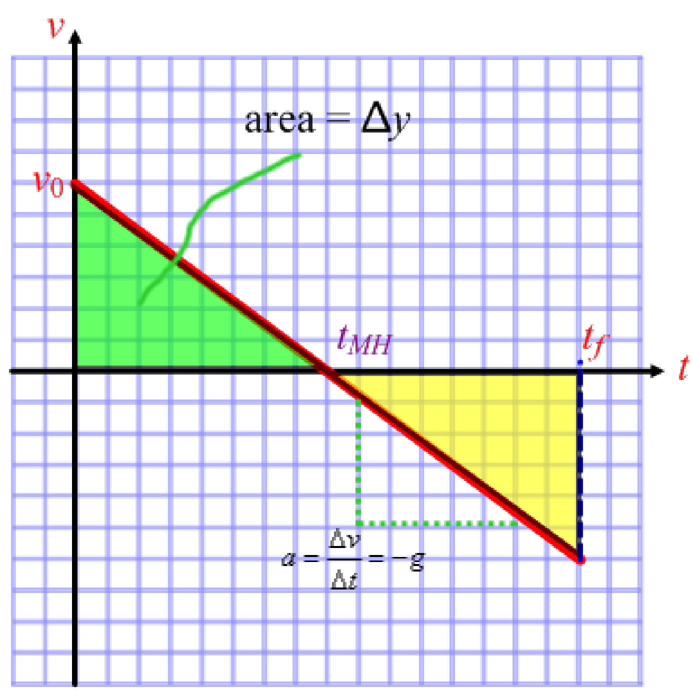


For an object launched upward that is in the air for a total time t_f, find

- the initial velocity v_0 at which it was launched and
- the maximum height h to which it rises.

Your answers should be equations for v_0 and h in terms of g and t_f only.



Since the object is in free fall, its velocity is a straight line starting at the initial speed and decreasing at a rate of g. Since the total displacement Δy is zero – the object ends up where it starts – the total area trapped by the velocity curve must also be zero. This means that the green and yellow triangles must be of equal area at the moment that $t = t_f$.

Where the velocity crosses the t-axis, the object is at rest – which, physically, is the moment it is at its greatest height. Geometrically, this must happen such that the two triangles are congruent. All told, this implies that the time to max height (t_{MH}) must be half the total time of flight (t_f):

$$t_{MH} = \tfrac{1}{2} t_f$$

We can find the slope between $t = 0$ and $t = t_{MH}$:

$$slope = \frac{\Delta v}{\Delta} = \frac{0 - v_0}{t_{MH} - 0} = -\frac{v_0}{t_{MH}}$$

Physically, though, the slope is the acceleration, which is known to be $-g$:

$$slope = -g$$

$$-\frac{v_0}{t_{MH}} = -g$$

$$v_0 = gt_{MH} = g\left(\tfrac{1}{2}t_f\right)$$

$$v_0 = \boxed{\tfrac{1}{2}gt_f}$$

But since t_{MH} is the time it reaches its maximum height, the area trapped by the velocity curve up to that point (the green triangle) must be the displacement at the highest point.

$$area = \tfrac{1}{2} base \times height = \tfrac{1}{2}\left(t_{MH}\right)\left(v_0\right) = \tfrac{1}{2}\left(\tfrac{1}{2}t_f\right)\left(\tfrac{1}{2}gt_f\right) = \tfrac{1}{8}gt_f^2$$

$$area = \Delta y = h$$

Together, these tell us

$$h = \boxed{\tfrac{1}{8}gt_f^2}$$

As an aside (not mentioned in the original problem), the symmetry of the velocity curve mandates that the object hit the ground with the same speed as the one with which it was launched.